Chapter 5
Lecture 3

Cross Section

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5.3 Cross-Section

Collision is when two bodies run into each other

Collision or **scattering** experiments are important to understand the nature of interaction between two particles.

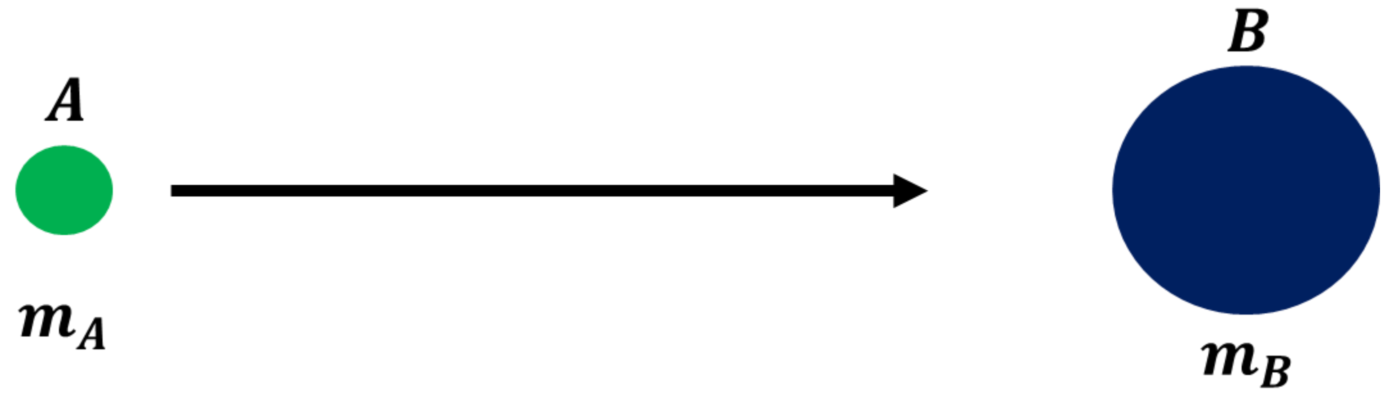
Nature of interacting Particles/bodies.

The most important in a collision or scattering is the cross-section of collision/Scattering.

To understand the cross section, we will start from very simple examples.

5.3 Cross-Section

Let us consider a point mass A is incident on a body B. The probability that ball/mass.



A will make collision with B depends on area of B .

If the area of B is large.

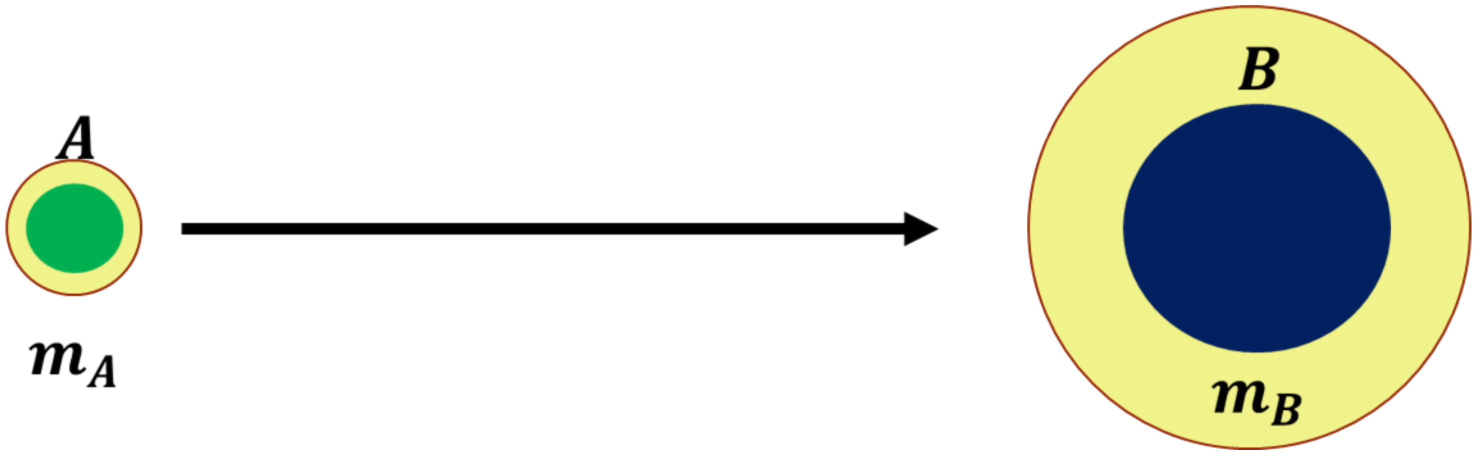
The chance that A will hit B is more or vice versa.

Therefore, the probability that A will collide with body B is more in case the cross-section area of B is large.

5.3 Cross-Section

If we replace balls by two charges.

Then the probability of collision will be more as charge particles can be scattered without physical collision.



Therefore,

The cross-section of B will be the area of an imaginary disc associated with B.

A more convenient way to express the probability of scattering is cross-section

5.3 Cross-Section

In collision experiment of microparticles (subatomic)

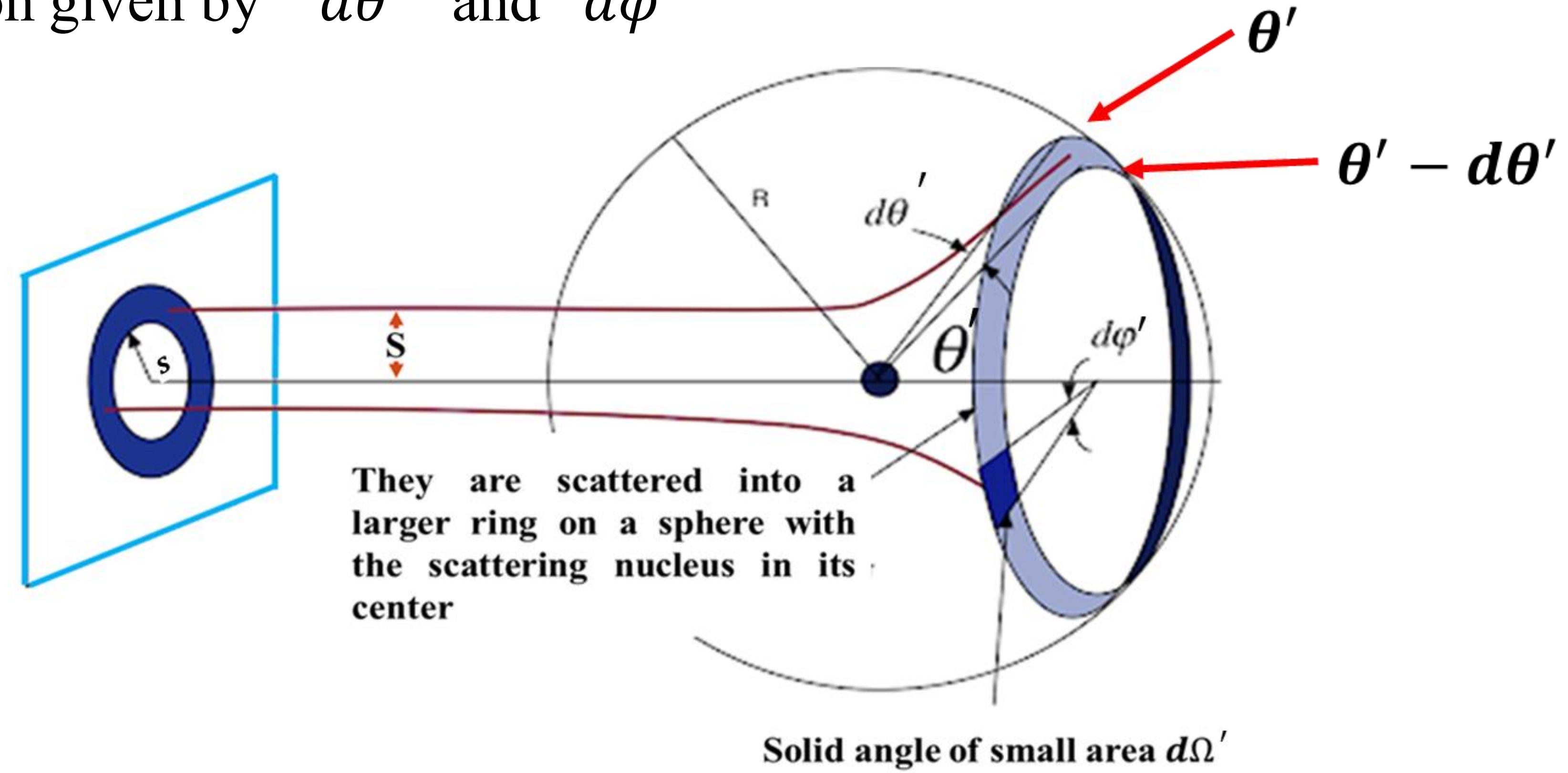
- Study individual particles through the scattering process is not possible.
- Particles are identical and cannot be distinguished.
- The concept of probability is introduced in terms of the cross-section.

Cross-section can be defined as the measure of probability that collision will take place when a single particle is bombarded on a target having one target particle per unit volume.

5.3 Differential Cross-Section in Canter of Mass Coordinates

Let "N" be the number of particles incident per unit area per second on target particles which are at rest.

Let "dN" be the number of particles scattered per second in solid angle "$d\Omega'$" along the direction given by "$d\theta'$" and "$d\varphi'$"



5.3 Differential Cross-Section in Canter of Mass Coordinates

The No. of scattered particles in solid angle "$d\Omega'$" will be proportional to the intensity of the incident particles and the magnitude of solid angle

$$dN \propto N d\Omega'$$

$$dN = \sigma(\theta') N d\Omega'$$

The constant is known as differential cross section

$$\sigma(\theta') = \frac{1}{N}\frac{dN}{d\Omega'}$$

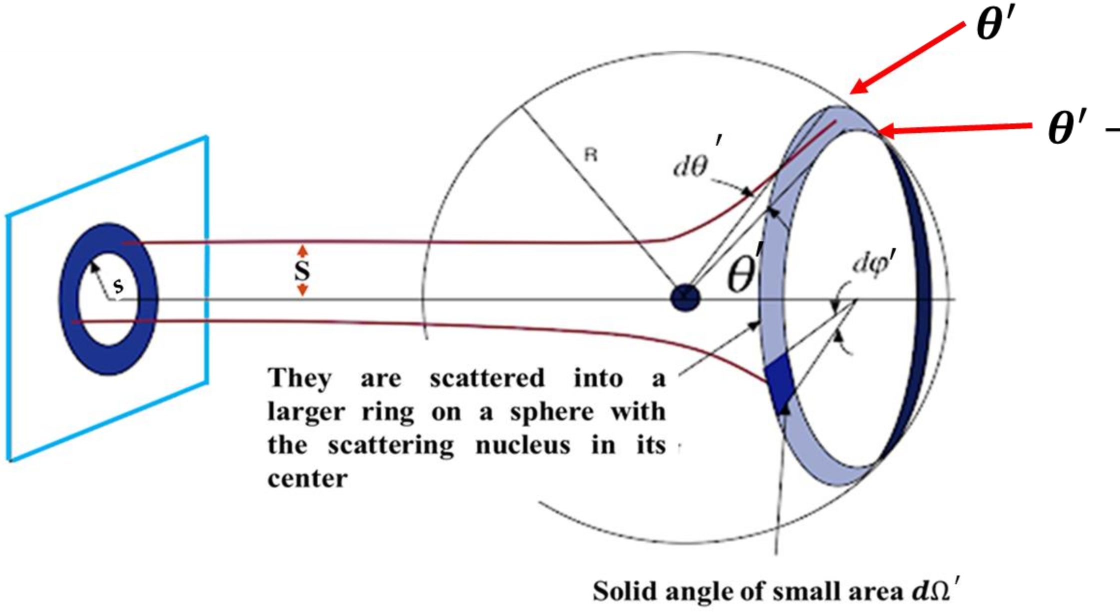
Where solid angle is $$d\Omega' = \sin\theta' \, d\theta' d\varphi'$$

For Azimuthal symmetry $$\int_0^{2\pi} d\varphi' = 2\pi$$

$$\Rightarrow d\Omega' = 2\pi \sin\theta' \, d\theta'$$

5.3 Differential Cross-Section in Canter of Mass Coordinates

- The particles with impact parameter "s" are scattered through angle "θ'"
- Those having impact parameter "$s + ds$" will be scattered through smaller angle "$\theta' - d\theta'$"
- Where all the particles in ring having radius between "s" and "$s + ds$" and area $2\pi s ds$ will be scattered in solid angle.
- For N is the total particles in the beam of area $A = \pi r^2$ (say: r is the radius of the circular beam), particles on surface of the ring between "s" and "$s + ds$" and area $2\pi s ds$ will be



$$dN = 2\pi s ds N$$

5.3 Differential Cross-Section in Canter of Mass Coordinates

Therefore,

$$dN = 2\pi s ds N = dN = \sigma(\theta')N d\Omega'$$

$$\Rightarrow 2\pi s ds N = \sigma(\theta')N(2\pi \sin\theta' \, d\theta')$$

$$\Rightarrow 2\pi s ds N = 2\pi N \sigma(\theta') \sin\theta' \, d\theta'$$

$$\Rightarrow s ds = \sigma(\theta') \sin\theta' \, d\theta'$$

$$\Rightarrow \sigma(\Omega') = \frac{d\sigma(\Omega')}{d\Omega'} = \frac{s}{\sin\theta'}\left|\frac{ds}{d\theta'}\right|$$

5.3 Differential Cross-Section in Canter of Mass Coordinates

Now the total Cross section will be

$$\boldsymbol{\sigma_T = \int \sigma(\theta')\, d\Omega'}$$

$$\boxed{\boldsymbol{\sigma_T = 2\pi \int \sigma(\theta')\ \sin\theta'\, d\theta'}}$$

This represent the total probability of particles scattered in all directions per unit intensity of incident bean per second.

5.4 Relation between cross section in C.M. and Lab. Coordinate system

Since the particles scattered in a solid angle $dN = 2\pi N\sigma(\theta_1)\sin\theta_1\, d\theta_1$ in laboratory system is equal to that scattered in corresponding solid angle $dN = 2\pi N\sigma(\theta')\ \sin\theta'\, d\theta'$ in C.M system. therefore

$$2\pi N\sigma(\theta')\ \sin\theta'\, d\theta' = 2\pi N\sigma(\theta_1)\sin\theta_1\, d\theta_1$$

$$\sigma(\theta') = \sigma(\theta_1)\frac{\sin\theta_1}{\sin\theta'}\frac{d\theta_1}{d\theta'}$$

Since the scattering angle in Lab. And C.M system are related by relation

$$\tan\theta_1 = \frac{\sin\theta'}{\frac{m_1}{m_2} + \cos\theta'}$$

If $m_1 = m_2$

$$\theta_1 = \frac{\theta'}{2}$$

5.4 Relation between cross section in C.M. and Lab. Coordinate system

$$\sigma(\theta') = \sigma(\theta_1)\frac{\sin\theta_1}{\sin\theta'}\frac{d\theta_1}{d\theta'} = \sigma(\theta_1)\frac{\sin\frac{\theta'}{2}}{\sin\theta'}\frac{\frac{d\theta'}{2}}{d\theta'}$$

$$\sigma(\theta') = \frac{1}{2}\sigma(\theta_1)\frac{\sin\frac{\theta'}{2}}{2\sin\frac{\theta'}{2}\cos\frac{\theta'}{2}}\frac{d\theta'}{d\theta'}$$

$$\boxed{\sigma(\theta') = \frac{1}{4\cos\frac{\theta'}{2}}\sigma(\theta_1) = \frac{1}{4\cos\theta_1}\sigma(\theta_1)}$$

Now if $m_1 \gg m_2 \Rightarrow \theta_1 = \theta'$

$$\sigma(\theta') = \sigma(\theta_1)\frac{\sin\theta_1}{\sin\theta'}\frac{d\theta_1}{d\theta'} = \sigma(\theta_1)\frac{\sin\theta'}{\sin\theta'}\frac{d\theta'}{d\theta'}$$

$$\boxed{\sigma(\theta') = \sigma(\theta_1)}$$

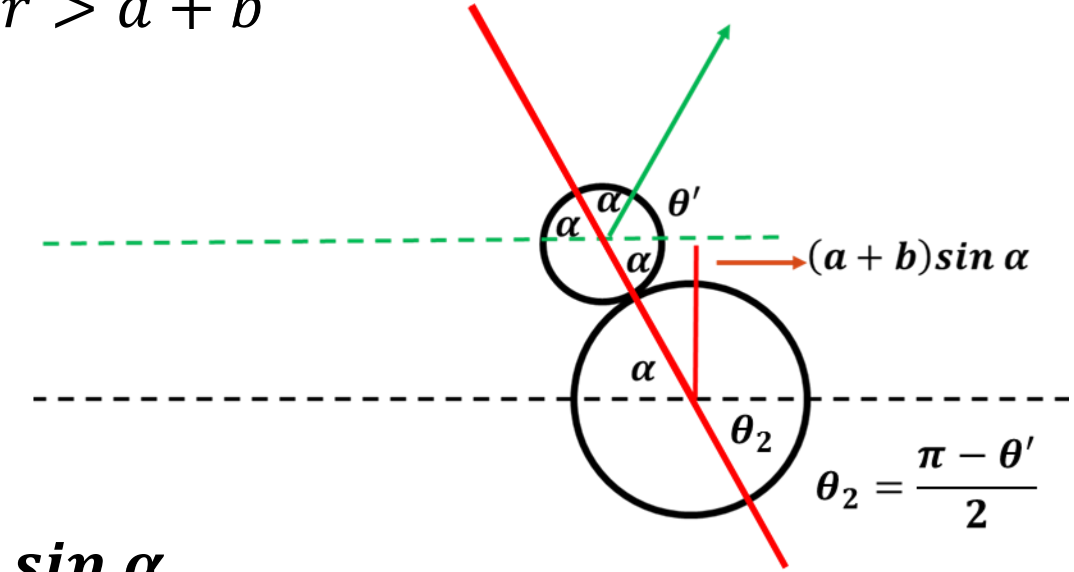
5.4 Effective Cross Section for scattering of particle with radius b by a sphere of radius a

Consider an elastic scattering of sphere having mass m_1 and radius b by a target sphere of mass m_2 and radius a in the C.M and Lab. System.

The law of force

$$V = \begin{cases} \infty & r < a + b \\ 0 & r > a + b \end{cases}$$

The incident sphere will get scattered after rebounding from the surface of the target sphere.



The impact parameter in this case is;

$$\boldsymbol{s = (a + b)\, sin\, \alpha}$$

$$\boldsymbol{s = (a + b)\, sin \left(\frac{\pi - \theta'}{2}\right)}$$

5.4 Effective Cross Section for scattering of particle with radius b by a sphere of radius a

$$s = (a+b)\cos\frac{\theta'}{2}$$

Differentiating above equation

$$\frac{ds}{d\theta'} = -\frac{1}{2}(a+b)\sin\frac{\theta'}{2}$$

$$\sigma(\theta') = \frac{s}{\sin\theta'}\left|\frac{ds}{d\theta'}\right|$$

$$\sigma(\theta') = \frac{(a+b)\cos\frac{\theta'}{2}}{\sin\theta'}\left|-\frac{1}{2}(a+b)\sin\frac{\theta'}{2}\right|$$

$$\sigma(\theta') = \frac{(a+b)^2\cos\frac{\theta'}{2}\sin\frac{\theta'}{2}}{2\,\sin\theta'}$$

5.4 Effective Cross Section for scattering of particle with radius b by a sphere of radius a

$$\sigma(\theta') = \frac{(a+b)^2}{4} \frac{2sin\,\frac{\theta'}{2} cos\,\frac{\theta'}{2}}{\sin\theta'}$$

$$\sigma(\theta') = \frac{(a+b)^2}{4} \frac{\sin\theta'}{\sin\theta'}$$

$$\boxed{\sigma(\theta') = \frac{(a+b)^2}{4}}$$

Now the total scattering Cross section in C.M system

$$\sigma_{C.M} = \int \sigma(\theta')\, d\Omega'$$

$$\sigma_{C.M} = \int \frac{(a+b)^2}{4} d\Omega'$$

5.4 Effective Cross Section for scattering of particle with radius b by a sphere of radius a

$$\sigma_{C.M} = \frac{(a+b)^2}{4} \int_0^{\pi} 2\pi \ \sin\theta' \, d\theta'$$

$$\boxed{\sigma_{C.M} = \pi(a+b)^2}$$

Now in the lab system

If $m_1 = m_2$

$$\sigma(\theta') = \sigma(\theta_1) \frac{1}{4\cos\theta_1}$$

$$\sigma(\theta_1) = \sigma(\theta') \, 4\cos\theta_1$$

$$\sigma(\theta_1) = \frac{(a+b)^2}{4} 4\cos\theta_1$$

5.4 Effective Cross Section for scattering of particle with radius b by a sphere of radius a

$$\sigma(\theta_1) = (a+b)^2 \cos\theta_1$$

Integrating above equation

$$\sigma(\theta_1)_T = \int \sigma(\theta_1)\, d\Omega = 2\pi(a+b)^2 \int_o^{\frac{\pi}{2}} \sin\theta_1 \cos\theta_1\, d\theta_1$$

$$\frac{1}{2}$$

$$\sigma(\theta_1)_T = 2\pi(a+b)^2 \int_o^{\frac{\pi}{2}} \sin\theta_1 \cos\theta_1\, d\theta_1$$

$$\sigma(\theta_1)_T = \pi(a+b)^2$$

5.4 Effective Cross Section for scattering of particle with radius b by a sphere of radius a

If $m_1 \ll m_2$ $$\boldsymbol{\sigma(\theta') = \sigma(\theta_1) = \frac{(a+b)^2}{4}}$$

$$\boldsymbol{\sigma(\theta_1)_T = \int \sigma(\theta_1)\, d\Omega = \frac{(a+b)^2}{4}\int d\Omega}$$

$$\boldsymbol{\sigma(\theta_1)_T = 2\pi\frac{(a+b)^2}{4}\int_o^{\frac{\pi}{2}} \sin\theta_1\, d\theta_1}$$

$$\boldsymbol{\sigma(\theta_1)_T = \frac{\pi}{2}(a+b)^2\int_o^{\frac{\pi}{2}} \sin\theta_1\, d\theta_1 = \frac{\pi}{2}(a+b)^2}$$

If $m_1 \ll m_2$ mean $a \gg b$ $$\boldsymbol{\sigma(\theta_1)_T = \frac{\pi}{2}a^2}$$